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# BI – IDEALS AND QUASI – IDEALS OF BCI ALGEBRA

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## ABSTRACT
In this Paper, we introduce the idea of Bi – ideals and Quasi ideals in BCI algebra. Also, we discuss the relationship between a Bi – ideal and a Quasi ideal and distinguish some properties of Bi – ideals and Quasi ideals in BCI algebra.

**KEYWORDS:** BCI algebra, ideal, Bi – ideal, Quasi ideal.

## 1. INTRODUCTION
The notion of BCI (BCK) – algebras was first initiated by Imai and Iseki in 1966 as a generalization of the concept of set – theoretic difference and prepositional calculus. It turns out that Abelian groups are a special case of BCI algebras. Y.B.Jun [1] deal with various results on ideals of BCK algebras. The impression of Bi – ideal for semi groups was interrupted by Good and Hughes.[4] Subsequently, the conception of Bi – ideals in associative rings were suggested by Lajos and Szasz.[5] T.Tamizh Chelvam et al. [2] innovated certain concepts on Bi – ideals of near rings. An interesting special case of Bi – ideals is given by the Quasi ideal of O. Steinfeld [6] in some characterizations of groups and division ring. I. Yakabe [3] constituted several properties on Quasi ideals in near rings.

This concept motivates to construct Bi-ideals and Quasi ideals in BCI algebra and discuss some basic results, examples and properties.

## 2. PRELIMINARIES
We recall several basic definitions which are needed for the evolution of this paper.

**DEFINITION 2.1:**

An algebra (X,*, 0) of type (2, 0) is called a BCI algebra if the following conditions are satisfied:
- i. $((x * y) * (x * z)) * (z * y) = 0$
- ii. $(x * (x * y)) * y = 0$
- iii. $x * x = 0$
- iv. $x * y = 0 \text{ and } y * x = 0 \text{ imply } x = y, \forall x, y, z \in X.$

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**EXAMPLE 2.2:**

Let X = {0, a, b, c} be a BCI algebra with the following cayley table:

| * | 0 | a | b | c |
|---|---|---|---|---|
| 0 | 0 | a | b | c |
| a | a | 0 | c | b |
| b | b | c | 0 | a |
| c | c | b | a | 0 |

(i.e) (X, $*$, 0) is a BCI algebra.

**DEFINITION 2.3:**

Let A and B be two non-empty subsets of X. We define two types of products:

i. $AB \ = \{ab / a \in A, b \in B\}$
ii. $A * B = \ \{a\,(a' + b) - aa' /\ a \in A, b \in B\}$

**DEFINITION 2.4:**

A non-empty subset S of a BCI algebra X is called a BCI subalgebra if $x * y \in S, \forall\, x, y \in S$.

**DEFINITION 2.5:**

A subset A of a BCI algebra X is called a BCI-ideal if it satisfies the following axioms:

i. 0 ∈ A
ii. $y * x \in A$ *and* $x \ \in A$ *imply* $y \in A$ $(resp. x * y \ \in A), \forall\, x, y \ \in X.$

**DEFINITION 2.6:**

A BCI algebra X is called Zero symmetric, if x.0 = 0, ∀ x∈X.

**DEFINITION 2.7:**

A BCI algebra X is said to be regular if every element of X is regular. (i.e.) for every a ∈ X, there exists b ∈ X such that a = aba.

**DEFINITION 2.8:**

A subgroup M of a BCI algebra X is said to be a BCI sub algebra if MM ⊆ M.

**DEFINITION 2.9:**

A subgroup M of a BCI algebra X is called a left (right) X- BCI subalgebra of X if XM ⊆ M (MX ⊆ M).

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**DEFINITION 2.10:**

An element a of a BCI algebra X is said to be idempotent if $a^2 = a$.

**DEFINITION 2.11:**

A BCI algebra X is called idempotent if $X = X^2$.

**DEFINITION 2.12:**

Let A be a set. $\mathcal{M} \subseteq \wp(A)$ (Where $\wp(A)$ denote the power set of A) is said to be a Moore-system on A ⇔

i. A ⊆ M.
ii. For any set I, $(\forall\, i \in I: M_i \in M) \Rightarrow \bigcap_{i \in I} M_i \in M$.

## 3. BI – IDEALS ON BCI ALGEBRA

**DEFINITION 3.1:**

A subalgebra B of a BCI algebra X is called a **Bi – ideal** of X if BXB ∩ (BX)*****B ⊆ B. In case of zero symmetric, BXB ⊆ B.

**EXAMPLE 3.2:**

Let X = {0, 1, a, b, c} be a BCI algebra with the following cayley table:

| * | 0 | 1 | a | b | c |
|---|---|---|---|---|---|
| 0 | 0 | 0 | a | b | c |
| 1 | 1 | 0 | a | b | c |
| a | a | a | 0 | c | b |
| b | b | b | c | 0 | a |
| c | c | c | b | a | 0 |

Clearly, B = {0, a} is a Bi – ideal of a BCI algebra X.

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## 3.1 MAIN RESULTS ON BI – IDEALS

**Proposition: 3.1.1**

The set of all bi – ideals of a BCI algebra X forms a Moore system on X.

**Proof:**

Let $B_i$ $\{ i \in I\}$ be a set of Bi – ideals of X. Let B = $\bigcap_{i \in I} B_i$.
Then BXB ∩ (BX) ∗ B ⊆ $B_iXB_i$ ∩ $(B_iX)$ ∗ $B_i$ ⊆ $B_i$, for every $i \in I$.
⇒BXB ∩ (BX) ∗ B ⊆ B.
Therefore, B is a Bi – ideal of X.

**Proposition: 3.1.2**

If B is a Bi – ideal of a BCI algebra X and S is a BCI sub algebra of X, then B ∩ S is a Bi – ideal of S.

**Proof:**

Since B is a Bi – ideal of X, BXB ∩ (BX) ∗ B ⊆ B. Let L = B ∩ S.
Then LSL ∩ (LS) ∗ L = $(B\cap S)$ S $(B\cap S) \cap \big((B \cap S)S\big) * (B \cap S)$
⊆ BSB ∩ S ∩ (BS) ∗ B ⊆ B∩S = L.
Therefore, L is a Bi – ideal of S.

**Proposition: 3.1.3**

Let X be a zero symmetric BCI algebra. A subalgebra B of X is a
Bi – ideal if and only if BXB ⊆ B.

**Proof:**

For a subalgebra B of X, if BXB ⊆ B, then B is a Bi – ideal of X.
Conversely,
Assume B is a Bi – ideal of X. We have BXB ∩ (BX) ∗ B ⊆ B.
Since X is zero symmetric, XB ⊆ X ∗ B, we get
BXB = BXB ∩ BXB ⊆ BXB ∩ (BX) ∗ B ⊆ B.
Hence BXB ⊆ B.

**Proposition: 3.1.4**

Let X be a zero symmetric BCI algebra. If B is a Bi – ideal of X,
then Bx and x'B are Bi – ideals of X where x, x' ∈ X and x' is distributive element in X.

**Proof:**

Clearly, Bx is a subalgebra of X and Bx X Bx ⊆ B X Bx ⊆ Bx,
We get, Bx is a Bi – ideal of X. Again x'B is a subalgebra of X.
Since x' is distributive in X and x'B X x'B ⊆ x'BXB ⊆ x'B.
Thus, x'B is a Bi – ideal of X.

## 4. QUASI – IDEALS ON BCI ALGEBRA

**DEFINITION 4.1:**

A subalgebra Q of a BCI algebra X is called a **Quasi – ideal** of X if
QX ∩ XQ ∩ X*****Q ⊆ Q. In case of zero symmetric, QX ∩ XQ ⊆ Q.

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**EXAMPLE 3.2:**

Let X = {0, 1, a, b, c} be a BCI algebra with the following cayley table:

| * | 0 | 1 | a | b | c |
|---|---|---|---|---|---|
| 0 | 0 | 0 | a | b | c |
| 1 | 1 | 0 | a | b | c |
| a | a | a | 0 | c | b |
| b | b | b | c | 0 | a |
| c | c | c | b | a | 0 |

Clearly, Q = {0, a, b, c} is a Quasi – ideal of a BCI algebra X.

**4.1 MAIN RESULTS ON QUASI – IDEALS**

**Proposition: 4.1.1**

The set of all Quasi – ideals of a BCI algebra X forms a Moore system on X.

**Proof:**

Let $Q_i$ { $i \in I$} be a set of Quasi – ideals of X. Let Q $= \bigcap_{i \in I} Q_i$.
Then QX ∩ XQ ∩ X∗ Q ⊆ $Q_i$X ∩ X$Q_i$ ∩ X∗ $Q_i$ ⊆ $Q_i$, for every $i \in I$.
Hence QX ∩ XQ ∩ X*Q ⊆ Q.
Therefore, Q is a Quasi – ideal of X.

**Proposition: 4.1.2**

If Q is a Quasi – ideal of a BCI algebra X and S is a BCI sub algebra of X, then Q ∩ S is a Quasi – ideal of S.

**Proof:**

Since Q is a Quasi – ideal of X, QX ∩ XQ ∩ X*Q ⊆ Q.
Clearly, Q ∩ S is a subalgebra of S. Let T = Q ∩ S.
(i.e.) To prove: T is a Quasi – ideal of S.

$$\text{Then } TS \cap ST \cap S * T = (Q\cap S)\, S \cap S\,(Q\cap S) \cap S * (Q \cap S)$$
$$\subseteq (Q \cap S)\, S \cap S\,(Q \cap S) \subseteq SS \subseteq S$$

and

$$TS \cap ST \cap S * T = (Q\cap S)\, S \cap S\,(Q\cap S) \cap S * (Q \cap S)$$
$$\subseteq QX \cap XQ \cap X * Q \subseteq Q$$

Therefore, TS ∩ ST ∩ S ∗ T ⊆ Q ∩ S = T ⇒ TS ∩ ST ∩ S ∗ T ⊆ T.
Hence T is a Quasi – ideal of S.

**Proposition: 4.1.3**

Let X be a zero symmetric BCI algebra. A subalgebra Q of X is a Quasi – ideal of X if and only if QX ∩ XQ ⊆ Q.

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**Proof:**

Let Q be a Quasi – ideal of X, then QX ∩ XQ ∩ X*Q ⊆ Q.
Since X is zero symmetric, XQ ⊆ X ∗ Q, we get
Now, QX ∩ XQ = (QX ∩ XQ) ∩ (QX ∩ XQ)
= QX ∩ (XQ ∩ QX) ∩ XQ
⊆ QX ∩ XQ ∩ X*Q ⊆ Q.
Conversely,
Assume QX ∩ XQ ⊆ Q.
Consider, QX ∩ XQ ∩ X*Q ⊆ QX ∩ XQ ⊆ Q.
Hence Q is a Quasi – ideal of X.

## 5. RELATIONSHIP BETWEEN BI – IDEALS AND QUASI – IDEALS

**Proposition: 5.1**

Every Quasi – ideal of a BCI algebra X is a Bi – ideal. But the Converse is not true.

**Proof:**

| * | 0 | 1 | a | b | c |
|---|---|---|---|---|---|
| 0 | 0 | 0 | a | b | c |
| 1 | 1 | 0 | a | b | c |
| a | a | a | 0 | c | b |
| b | b | b | c | 0 | a |
| c | c | c | b | a | 0 |

Let X = {0, 1, a, b, c} be a BCI algebra with the following cayley table:

It is easy to prove (X, ∗, 0) is a BCI algebra.
Clearly, B = {0, a} is a Bi – ideal.
But Q = {0, a} is not a Quasi – ideal.

## 6. BI – IDEALS WHICH ARE ALSO QUASI – IDEALS

**Proposition: 6.1**

Let B be a Bi – ideal of a BCI algebra X. If B is itself a regular BCI algebra, then any Bi – ideal of B is a Bi – ideal of X.

**Proof:**

Let A be a Bi – ideal of B. Since B is regular, for every a ∈ A ⊆ B,
a = aba, for some b ∈ B and so A ⊆ AB ∩ BA.
Hence AXA ⊆ (AB) X (BA) ⊆ A (BXB) A ⊆ ABA ⊆ A.
Thus, A is a Bi – ideal of B.

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**Proposition: 6.2**

Let B be a Bi – ideal of a BCI algebra X. If elements of B are regular, then B is a Quasi – ideal of X.

**Proof:**

Let c ∈ BX ∩ XB. Then c = ba = a'b', for some b, b' ∈ B and a, a' ∈ X.
Since B is regular, b = bgb, for some g ∈ B.
Then c = ba = (bgb) a = (bg) (ba) = (bg) (a'b') ∈ BXB ⊆ B.
(i.e.) XB ∩ BX is a Quasi – ideal of X.

**Corollary: 1** If B is a Bi – ideal and a regular BCI subalgebra of X, then any Bi – ideal of B is a Quasi – ideal of X as well as of B. If Q is a Quasi – ideal of X which is itself regular, then any Quasi – ideal of Q is also a Quasi – ideal of X.

**Proof:**

First to prove that any Bi – ideal of B is a Quasi – ideal of X as well as of B.
Given that B is a Bi – ideal and a regular BCI subalgebra.
Then BXB ⊆ B. Let A be any Bi – ideal of B. Then ABA ⊆ A.
Since B is regular, for every a ∈ A ⊆ B, a = aba, for some b ∈ B and so
A ⊆ AB ∩ BA.
Consider, AB ∩ BA ⊆ A (B ∩ B) A ⊆ ABA ⊆ A.
⇒ A is a Quasi – ideal of B.
Similarly, AX ∩ XA ⊆ (AB) X ∩ X (BA) ⊆ A (BX ∩ XB) A ⊆ ABA ⊆ A.
⇒ A is a Quasi – ideal of X.

Next to prove that any Quasi – ideal of Q is also a Quasi – ideal of X.
Given that Q is a Quasi – ideal of X and regular.
Since Q is a Quasi – ideal of X, QX ∩ XQ ⊆ Q.
Let P be a Quasi – ideal of Q.
Also, Q is regular, for every q ∈ P ⊆ Q, q = qrq, for some r ∈ Q.
Now, PX ∩ XP ⊆ (PQ) X ∩ X (QP) ⊆ P (QX ∩ XQ) P ⊆ PQP ⊆ P.
Thus, P is a Quasi – ideal of Q.

**Corollary: 2** A subalgebra M of a regular BCI algebra X is a Quasi – ideal if and only if M is a Bi – ideal of X.

**Proof:**

Assume M is a Quasi – ideal of X. Then MX ∩ XM ∩ X ∗ M ⊆ M.
Consider, (MXM) ∩ (MX) ∗ M = M (X ∩ X) M ∩ (MX) ∗ M
⊆ MX ∩ XM ∩ X ∗ M ⊆ M.
⇒ (MXM) ∩ (MX) ∗ M ⊆ M.
Hence M is a Bi – ideal of X.

Conversely,
Assume M is a Bi – ideal of X. Then (MXM) ∩ (MX) ∗ M ⊆ M.
Consider, MX ∩ XM ∩ X ∗ M ⊆ (MXM) ∩ X ∗ M
⊆ (MXM) ∩ (MX) ∗ M ⊆ M
⇒ MX ∩ XM ∩ X ∗ M ⊆ M.
Hence M is a Quasi – ideal of X.

**Corollary: 3** A subalgebra M of a regular BCI algebra A is a Quasi – ideal of

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A if and only if M satisfies the condition MAM ⊆ M.

**Proof:**

Assume M is a Quasi – ideal of A. Then MA ∩ AM ⊆ M.
Consider, MAM = M (A ∩ A) M ⊆ MA ∩ AM ⊆ M.
⇒ MAM ⊆ M.
Conversely,
Assume MAM ⊆ M.
Consider, MA ∩ AM ⊆ M (A ∩ A) M ⊆ MAM ⊆ M.
⇒ MA ∩ AM ⊆ M.
Hence M is a Quasi – ideal of A.

**THEOREM: 1**

Let X be a regular BCI algebra in which idempotent commute. Then every Quasi – ideal of X is idempotent.

**Proof:**

Let P be a Quasi – ideal of X and a ∈ P. Since P is a BCI subalgebra, $P^2$ ⊆ P. We have to prove P ⊆ $P^2$. (i.e.) a ∈ $P^2$. Since X is regular, a = axa, for every x ∈ X. Here, xa is an idempotent and xa is in centre of X.
By [7] Theorem 1, using $MX^2M$ ⊆ MX ∩ XM ⊆ M, we get $PX^2P$ ⊆ PX ∩ X P ⊆ P
⇒ a = (ax) a (xa) = (ax) (xa) a = ($ax^2a$) a ∈ ($PX^2P$) P⊆ $P^2$ ⇒ a ∈ $P^2$.
Hence P = $P^2$.
Therefore P is idempotent.

**Proposition: 6.3**

Let X be a regular BCI algebra in which every Quasi – ideal of X is idempotent. Then for left X – subalgebra L and right X – subalgebra R of X,
RL = R ∩ L ⊆ LR is true.

**Proof:**

Let A and B be two Quasi – ideals of X, then A ∩ B is also a Quasi – ideal of X. Since X is idempotent, A ∩ B is also idempotent.
Then A ∩ B = $(A ∩ B)^2$ ⊆ AB ∩ BA. On the other hand,
AB ∩ BA ⊆ AX ∩ XA ⊆ A and analogously, AB ∩ BA ⊆ B
⇒ A ∩ B = AB ∩ BA.
Since X – subalgebras are always Quasi – ideals, we have R ∩ L = RL ∩ LR,
but RL ⊆ R ∩ L and so RL = R ∩ L ⊆ LR holds.

**Preposition: 6.4**

Let L and R be left X – subalgebra and right X – subalgebra of a BCI algebra X. Then any subalgebra B of X such that RL ⊆ B ⊆ R ∩ L is a Bi – ideal of X.

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**Proof:**

For a subalgebra B of X with RL ⊆ B ⊆ R ∩ L, we have
BXB ⊆ (R ∩ L) X (R ∩ L) ⊆ RXL ⊆ RL ⊆ B ⇒ BXB ⊆ B.
Thus, B is a Bi – ideal of X.

## 7. ACKNOWLEDGEMENT

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